Improving Resource Utilization for Compositional Scheduling using DPRM Interface

Jaewoo Lee, Linh T.X. Phan, Sanjian Chen, Oleg Sokolsky, and Insup Lee
Introduction

- Compositional scheduling analysis
  - Workload: periodic tasks (p, e)
  - Resource model: component interface
Introduction

- Periodic Resource Model (PRM) $\Gamma = (\Pi, \Theta)$
  - At least $\Theta$ resources in every $\Pi$ time units
    - Supply Bound Function of $\Gamma$ (sbf$_\Gamma$) : minimum resource supply
  - Schedulability condition in PRM

\[ \text{(minimum possible) resource supply} \geq \text{(maximum possible) resource demand} \]

- Demand Bound Function of workload $W$ (dbf$_W$)

- Optimality
  - PRM $\Gamma$ for $W$ is bandwidth-optimal iff $\Theta/\Pi$ is the minimum and $\Gamma$ can schedule $W$
Practical Consideration

- **Optimal PRM**
  - Minimum bandwidth \((\Theta/\Pi)\) to schedule workload \(W\)
  - Optimal algorithm [1]: rational number in \(\Theta\).
  - But, \(\Theta\) should be integer multiple of the time slice

- **Ex:**
  - Optimal algorithm:
    - \((1,0.595)\)
  - Only integer:
    - \((1,1), (2,2), (3,2), (4,3), \ldots\)
  - Among all possible PRM, \((3,2)\) is optimal with integer

Contribution

- Efficient algorithm for optimal PRM with integer.
- Quantize overheads of optimal PRM with integer.
- Introduce Dual Periodic Resource Model (DPRM)
Efficient algorithm for optimal PRM with integer

- **NaïveSearch**: naïve algorithm for optimal PRM

For $\Pi = 1$ to $\Pi_{\text{max}}$

$$\Theta = \text{MinExec}(\Pi, W)$$

If $\Theta / \Pi < \text{minBW}$

$\text{minBW} = \Theta / \Pi$

$\Pi' = \Pi$, $\Theta' = \Theta$

EndIf

EndFor

**workload $W$**

**Designer**

$W = \{(10, 1), (12, 2), (15, 4)\}$ is used in the rest of the section

**MinExec**: compute the feasible minimum execution time for given workload and period

**Non-decreasing**

the upper bound of $\Pi$ to find optimal PRM?
Efficient algorithm for optimal PRM with integer

- Upper supply bound function ($\text{usbf}_\Gamma$):
  - Linear function which is the smallest upper-bound of $\text{sbf}_\Gamma(t)$

- USBF-schedulability condition: necessary condition
  - Min. resource supply of $\text{usbf}_\Gamma \geq \text{Max. resource demand of } W$

![Graph showing resource vs. time with specific points and lines](image_url)
Efficient algorithm for optimal PRM with integer

- Theorem 1 (Upper bound of period in NaïveSearch)
  - If $\Gamma_\beta$ is the min. bandwidth PRM for $W$ s.t. $\Pi \leq \beta$ and $\kappa$ is the bandwidth of $\Gamma_\beta$,
  - Then $\Pi_{opt} \leq \min_{t \in \text{CrT}_W} \frac{\kappa t - \text{dbf}_W(t)}{\kappa (1 - \kappa)}$.

Critical Times for $W$ (CrT$_W$)

- Sufficient subset for USBF-schedulability
- For any $\Gamma$ satisfying USBF schedulability,
  - intersection with usbf$_\Gamma(t)$ and dbf$_W(t)$
  - x coordinate of the points

In Ex. CrT$_W$ = \{15,60\}
Efficient algorithm for optimal PRM with integer

- **Theorem 1 (Upper bound of period in NaïveSearch)**
  - If $\Gamma_\beta$ is the min. bandwidth PRM for $W$ s.t. $\Pi \leq \beta$ and $\kappa$ is the bandwidth of $\Gamma_\beta$,
  
  - Then $\Pi_{\text{opt}} \leq \min_{t \in \text{Cr} T_W} \frac{\kappa t - \text{dbf}_W(t)}{\kappa(1 - \kappa)}$.

- **In Example**
  - $\text{Cr} T_W = \{15, 60\}$
  - $\Gamma_3 = (3, 2)$ until $\Pi = 3$
  - By Thm. 1, $\Pi_{\text{opt}} \leq 13.5$
    - Since optimal PRM is $(5, 3)$,
    - it satisfies Thm. 1.
FastSearch: efficient algorithm for optimal PRM

- Observation: $\Theta \leq \Pi$ in PRM ($\Pi$, $\Theta$)
- Search space can be reduced by iterating $\Theta$, instead of $\Pi$

MinExec: compute the feasible minimum execution time

MaxPeriod: compute the feasible maximum period for given workload and execution time
Efficient algorithm for optimal PRM with integer

- Theorem 3 (Upper bound of $\Theta$ in FastSearch)
  - If $\Gamma_\beta$ is the min. bandwidth PRM for $W$ s.t. $\Theta \leq \beta$ and $\kappa$ is the bandwidth of $\Gamma_\beta$,
  
  Then $\Theta_{opt} \leq \min_{t \in CrT_W} \frac{\kappa t - dbf_W(t)}{1 - \kappa}$.

- In Example
  - $CrT_W = \{15, 60\}$
  - $\Gamma_2 = (3, 2)$ until $\Theta = 2$
  - By Thm. 3, $\Theta_{opt} \leq 9$
    - Search space reduction $= \frac{2}{3}$
    - $\Pi_{opt} \leq 13.5$
Contribution

- Efficient algorithm for optimal PRM with integer
- Quantize overheads of optimal PRM with integer
- Introduce Dual Periodic Resource Model (DPRM)
Overhead of optimal PRM with integer

- Optimal PRM with rational number [1]
  - \((1, B)\) where \(B\) is utilization of the workload.
- Optimal PRM with integer
  - \((\Pi, \Theta)\) where \(\Pi, \Theta\) are integer and computed by FastSearch

Ex.

- \(W=\{(10,2)\}\).
- Rational number optimal PRM = \((1, 0.2)\), bandwidth= 0.2
- Integer optimal PRM = \((3, 1)\), bandwidth= 0.33
- Bandwidth overhead : 66%

Contribution

- Efficient algorithm for optimal PRM with integer
- Quantize overheads of optimal PRM with integer
- Introduce Dual Periodic Resource Model (DPRM)
Dual Periodic Resource Model (DPRM)

- DPRM: contains two PRMs
  - $\Omega = (\Gamma_1, \Gamma_2) = ((\Pi_1, \Theta_1), (\Pi_2, \Theta_2))$
  - $\Theta_1$ unit in every $\Pi_1$ time units, additionally $\Theta_2$ unit in every $\Pi_2$
  - Bandwidth = $BW(\Gamma_1) + BW(\Gamma_1)$

Ex. resource supply of DPRM ((6,1), (8,1))

\[ \Omega = ((6,1), (8,1)) \]
Can DPRM reduce the overhead?

Ex.

- \( W = \{(7, 1), (11, 3), (13, 2)\} \)
- Optimal PRM = (3, 2), bandwidth = 0.667
- Optimal DPRM = \{(2, 1), (7, 1)\}, bandwidth = 0.643
- Bandwidth reduction = 3.73%
Dual Periodic Resource Model (DPRM)

- **DualSearch**: Algorithm for optimal DPRM

  - workload \( W \)

  - By Thm.1

  - For \( \Pi_1 = 1 \) to \( \Pi_1^{\text{max}} \)
    - For \( \Theta_1 = 1 \) to \( \Theta_1^{\text{max}} \)
      - \( \Gamma_1 = (\Pi_1, \Theta_1) \)
      - \( \Gamma_2 = \text{getResModel} (\Gamma_1, W) \)
      - If \( BW(\Gamma_1) + BW(\Gamma_2) < \text{minBW} \)
        - \( \text{minBW} = BW(\Gamma_1) + BW(\Gamma_2) \)
        - \( \Gamma_1' = \Gamma_1, \Gamma_2' = \Gamma_2 \)
      - EndIf
    - EndFor
  - EndFor

  - \( \Omega' = (\Gamma_1', \Gamma_2') \)

  - \( \text{getResModel} \) : compute the feasible minimum-bandwidth PRM \( \Gamma_2 \) for given workload \( W \) and PRM \( \Gamma_1 = (\Pi_1, \Theta_1) \)

- By FastSearch

- calculate remaining demand

- \( W' \)

- find optimal PRM for \( W' \)

- \( \Gamma_2 = (\Pi_2, \Theta_2) \)
Simulation

First 10 workloads of 200 random workload

Workload = 3 task

PRM with integer

PRM with rational number
## Simulation

<table>
<thead>
<tr>
<th>The number of tasks in workload</th>
<th>% of DPRM with smaller bandwidth</th>
<th>Maximum bandwidth reduction</th>
<th>PRM overhead (A)</th>
<th>DPRM overhead (B)</th>
<th>(A)/(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>77%</td>
<td>12.5%</td>
<td>3.22%</td>
<td>1.25%</td>
<td>2.58</td>
</tr>
<tr>
<td>4</td>
<td>81%</td>
<td>15.86%</td>
<td>3.27%</td>
<td>1.21%</td>
<td>2.84</td>
</tr>
<tr>
<td>5</td>
<td>86%</td>
<td>9.09%</td>
<td>1.81%</td>
<td>0.46%</td>
<td>3.93</td>
</tr>
</tbody>
</table>

Each 200 random workload
Conclusion

- **Problem**
  - Existing algorithm [1] use rational number
  - In NaïveSearch algorithm
    - designer choose search space (not optimal)

- **FastSearch** algorithm in PRM
  - Consider **integer** parameters
  - Present a **safe upper bound** of search space for optimal resource model.

- **Dual periodic resource model**
  - Achieve **smaller bandwidth than single periodic resource model** in over 77% of workloads in the simulation

Thank you
Time Complexity of Each Algorithm

- **NaïveSearch with Theorem 1**: $O(\left(LCM_W\right)^2 \cdot \min_{P_i \in W} P_i)$.

- **FastSearch**: $O(\left(LCM_W\right)^2 \cdot \min_{P_i \in W} P_i)$.
  - From Thm. 3 and Thm. 1,
    - Thm. 3 is $\kappa$ times faster than Thm. 1.

- **DualSearch**: $O((LCM_W)^4 \cdot (\min_{P_i \in W} P_i)^3)$
Backup: Triple Periodic Resource Model

- **Complexity**
  - grows fast

- **Bandwidth reduction**
  - is not significant due to significant reduction in dual model.

- **Future work**
  - Reduce complexity of DualSearch
  - Find more efficient algorithm to find optimal DPRM
Time Complexity of Each Algorithm

- TriplePeriodicResourceModel:
  \[O((\text{LCM}_W)^6 \cdot (\min_{P_i \in W} P_i)^5)\]